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Dr. Edwin Bishop has made substantial progress in his work on the internal temperature distributions in idealized models of the planets Jupiter and Saturn. A number of technical problems with the Runge-Kutta type routine for two-dimensional integrations of a general non-linear heat-conduction equation have been overcome, and the routine has been found to be capable of handling integrations employing an actual model of the planet Jupiter, that of W.C. DeMarcus. It turns out that, whenever locally the quantity $D\,\delta t/\delta r^2 \lesssim 0.5$, where D is the local value of diffusivity and δt and δr are radial and temporal intervals, this routine, as one might expect from the theory of the process applied to the linear case, generates divergent results.

For the Jupiter case, thermal histories can be computed (on the IBM system 360) in runs of reasonable duration provided the central core is omitted from the thermal profile. The existence of such a dense core is conjectural, and in DeMarcus' model, based on extrapolated equations of state for hydrogen with some helium admixture, was added primarily to yield the correct total mass consistent with constraints imposed by the observed centrifugal flattening. Therefore, a slight inner-boundary-condition modification was incorporated, based upon the observation that one may substitute for the temperature profile of any portion of the body, from the center $r = 0$ to some radius R , $0 \leq R \leq R_s$, R_s being the planet's radius, a single function of the time. Work is proceeding on Jupiter models on this basis.

The cutoff R is taken at the core boundary, where with an assumed law of metallic conduction $K = a\rho^b$, $b > 1$, following DeMarcus, the diffusivity becomes too large to meet the required stability criterion without a ten-fold decrease in time step that renders integration-run duration prohibitively long. In the case of the Saturn model of DeMarcus the core region extends to an appreciable fraction of the radius, and in any event it would be desirable to obtain temperature profiles of these core regions if evaluation in terms of hypothetical phase transitions should be wanted in the future. Therefore the potentially much faster implicit or Crank-Nicholson technique has been programmed for the problem. In the linear case, the method has no stability restriction on the size of δt, which can be adjusted for optimum integration-run duration over a fixed model-time interval, balancing the number of convergence iterations per step for a fixed accuracy against the model-time interval per step.

Preliminary results indicate that in the present non-linear context, the method does obviate the necessity to keep $D\delta t/\delta r^2 \lesssim 0.5$ and is at least comparably fast, with equal time steps, with the R.-K. process.

The progress of work was delayed by the change-over to the IBM 360/95 at the Goddard Space Institute in New York, should ultimately benefit greatly from the faster machine, especially with the Runge-Kutta integrations, where run durations, while not prohibitive if core regions are omitted, are significant enough to limit the number of systematic model-parameter variations that can be investigated.